642 *KSME International Journal, Vol. 12, No. 4, pp. 642~658, 1998*

Dynamic Modeling and Cooperative Control of a Redundant Manipulator Based on Decomposition

Jang Myung Lee*
(Received July 24, 1997)

This paper demonstrates the use of a redundant manipulator to execute multiple tasks specified at different points on the manipulator. This is accomplished by decomposing a redundant arm at an intermediate arm location, called "elbow", into two non-redundant local arms, referred to as the "basearm" and the forearm. This decomposition transforms a redundant arm into a "serially linked dual-arm system," where the cooperation between the basearm and the forearm is carried out through the task distribution and the elbow control. To distribute a given task to individual local arms according to their dynamic capabilities, the Cartesian space model of a serially linked dual-arm system is derived using Lagrangian mechanics. The Cartesian space dynamic model enables us to quantify the dynamic capabilities of individual arms based on two hyper-ellipsoids : the Cartesian Force Ellipsoid (C. F. E) representing the range of Cartesian forces due to the unit norm of joint torques, and Cartesian Acceleration Ellipsoid (C. A. E) representing the range of Cartesian accelerations due to the unit norm of Cartesian forces. In addition to the local dynamic characteristics, the global task requirements such as singularity avoidance, joint torque limit avoidance, motion generation efficiency, and accurate motion control, are improved by elbow control. Elbow control can also be used to execute a subtask at the elbow, for example, obstacle avoidance.

Key Words : Redundant Manipulator, Decomposition, Dynamic Capabilities, Task Distribution

1. Introduction

In recent designs of robotic manipulators, a number of extra joints are added to provide dexterity and versatility as well as a large workspace for the manipulator (Chang, 1986 ; Hollerbach and Suh, 1987 ; Lunde et al., 1987 ; Sharon et al., 1988 ; Book, 1985 ; Maciejewski and Klein, 1985). With redundancy, there exists multiple sets of joint motions which correspond to a desired Cartesian motion (task). The problem of resolving the redundancy is equivalent to the problem of finding the optimal set of joint motions which generates the desired Cartesian motion. Many methods have been developed for the utilization of redundancy. Most researchers have used the generalized inverse of the Jacobian matrix with a general performance function as a central tool in redundancy resolution (Lee and Lee, 1988, 1990 ; Yoshikawa, ; Hollerbach and Suh, 1987 ; Lunde et al., 1987 ; Maciejewski and Klein, 1985 ; Nakamura and Hanafusa, 1987 ; Li and Satry, 1988), or they have defined an extended Jacobian to obtain the corresponding joint velocities.

In a previous paper (Lee and Lee, 1990), we handled this redundancy resolution problem by treating a redundant manipulator as two serially linked manipulators, each being non-redundant

* Department of Electronics Engineering, Research Institute of Computer, Information and Communication, Pusan National University Pusan, 609-735, Korea

and defined by a predetermined intermediate point. The paper has shown (1) how the task at the HAND (end-effector of the forearm) and the task at the ELBOW (end-effector of the basearm) can be achieved simultaneously, and (2) how the basearm (B-ARM) can cooperate with the forearm (F-ARM) for the task at the HAND. This scheme provides a generalized concept of macro/micro manipulator system (Sharon et al., 1988). That is, while there are two permanent local manipulators in the macro/micro manipulator system, according to our scheme, the pre-determined intermediate point divides a redundant manipulator into two local arms dynamically. The scheme is both promising and elegant in terms of the kinematic control of a redundant manipulator. However, since the task is represented in terms of Cartesian velocities, the dynamic characteristics of each local arm have not been considered in achieving the task specified at the hand efficiently.

In this paper, we present a new concept of redundancy resolution in the Cartesian acceleration domain for a decomposed-redundant manipulator. First, we model a decomposed-redundant manipulator as the equivalent kinetic energy and potential energy matrices of the two local arms, an F-ARM and a B-ARM, in Cartesian space. The dynamic equations of the redundant manipulator are derived by using this dynamic model, in which the interacting force between the local arms -an inherent term in the serially linked manipulator system-is explicitly represented. Based on these dynamic equations, the task (represented in terms of the Cartesian acceleration) is distributed to each local arm according to each arm's dynamic characteristics. With the local optimal task distribution, the configuration of the decomposed-redundant manipulator can be optimized at the beginning of each task segment by elbow control (Lee and Lee, 1990) to satisfy the global task requirements.

2. Preliminary

2.1 Kinematics of a decomposed-redundant manipulator

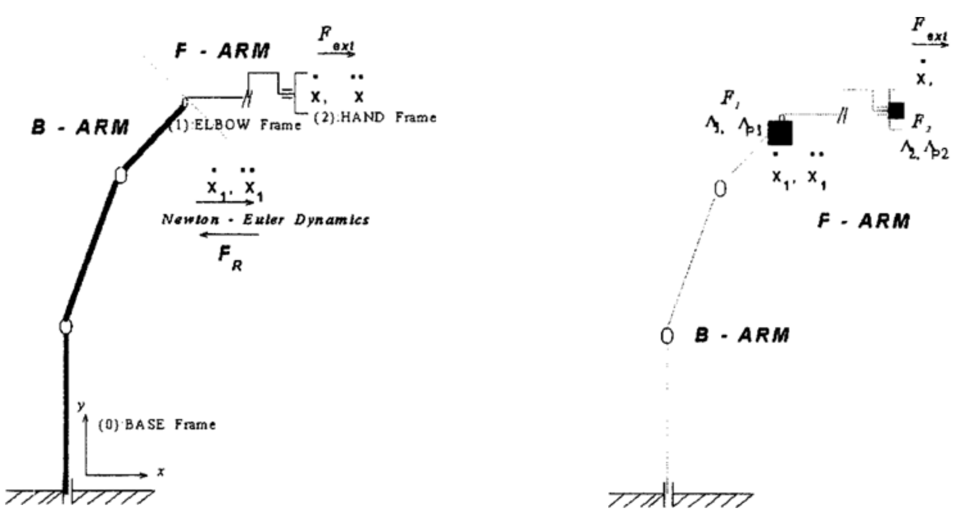
Figure 1(a) illustrates a decomposed-redundant arm represented as a serially linked manipulator system composed of a B-ARM and a F-ARM. The basic frames involved in the system are designated as follows :

BASE : the base frame of the system or equivalently, the base frame of the B-ARM ;

ELBOW : the end frame of B-ARM or equivalently, the base frame of the F-ARM ;

HAND : the end-effector frame of F-ARM.

Let us first represent the joint velocity vector of a



(a) A serially linked manipulator (b) Cartesian model

Fig. 1 Dynamic model of a decomposed-redundant manipulator redundant arm, $\dot{\theta}\in R^n$, as the combination of the joint velocity vector of the B-ARM, $\dot{\theta}_b\in R^b$, and the F-ARM, $\dot{\theta}_f\in R^f$ as follows :

$$\dot{\theta}=\begin{bmatrix}\dot{\theta}_b\\ \dot{\theta}_f\end{bmatrix}. \tag{1}$$

The elbow motion, $\dot{x}_1$, generated by the B-ARM can be described with reference to BASE by using the Jacobian of the B-ARM, ${}^0J_1$:

$$\dot{x}_1=\begin{bmatrix}{}^0v_1\\ {}^0\omega_1\end{bmatrix}=\begin{bmatrix}{}^0J_1^v\\ {}^0J_1^\omega\end{bmatrix}[\dot{\theta}_b]\,(={}^0J_1\dot{\theta}_b) \tag{2}$$

where ${}^0v_1$ and ${}^0\omega_1$ respectivly represent the linear and angular velocities of ELBOW in Cartesian space with respect to BASE.

Similarly, the hand motion, ${}^1\dot{x}_2$, generated by the F-ARM can be described with respect to ELBOW by using the Jacobian of the F-ARM, ${}^1J_2$:

$${}^1\dot{x}_2=\begin{bmatrix}{}^1v_2\\ {}^1\omega_2\end{bmatrix}=\begin{bmatrix}{}^1J_2^v\\ {}^1J_2^\omega\end{bmatrix}[\dot{\theta}_f]. \tag{3}$$

Based on (1-3), the hand motion, $\dot{x}\in R^m$, with respect to BASE can be expressed in terms of $\dot{\theta}_b$ and $\dot{\theta}_f$ as follows :

$$\dot{x}=\begin{bmatrix}{}^0v\\ {}^0\omega\end{bmatrix}=\begin{bmatrix}{}^0J_1^v\dot{\theta}_b+{}^0J_1^\omega\dot{\theta}_b\times P+{}^0R_1{}^1J_2^v\dot{\theta}_f\\ {}^0J_1^\omega\dot{\theta}_b+{}^0R_1{}^1J_2^\omega\dot{\theta}_f\end{bmatrix}$$

$$={}^0J_b\dot{\theta}_b+{}^0J_f\dot{\theta}_f \tag{4a}$$

$$=\begin{bmatrix}{}^0v_1+{}^0\omega_1\times P+{}^0R_1{}^1v_2\\ {}^0\omega_1+{}^0R_1{}^1\omega_2\end{bmatrix} \tag{4b}$$

$$=a\dot{x}_1+\dot{x}_2 \tag{4c}$$

where ${}^0J_b=\begin{bmatrix}{}^0J_1^v-\bar{P}{}^0J_1^\omega\\ {}^0J_1^\omega\end{bmatrix}$, ${}^0J_f=\begin{bmatrix}{}^0R_1{}^1J_2^v\\ {}^0R_1{}^1J_2^\omega\end{bmatrix}$, $a=\begin{bmatrix}T & -\bar{P}\\ 0 & T\end{bmatrix}$, $\dot{x}_2=\begin{bmatrix}{}^0R_1 & 0\\ 0 & {}^0R_1\end{bmatrix}{}^1\dot{x}_2$, P is the position vector from ELBOW to HAND, $\bar{P}\in R^{3\times3}$ is the cross-product operator of P (i. e., $\bar{P}=P\times$), ${}^0R_1$ the rotational transformation matrix from BASE to ELBOW, and T is a 3×3 identity matrix. (4c) indicates that the hand velocity can be represented explicitly in terms of the Cartesian velocities of B-ARM and F-ARM.

Consequently, the hand acceleration, $\ddot{x}$, with reference to BASE can be derived by taking the time derivatives of both sides of (4b) :

$$\ddot{x}=\begin{bmatrix}{}^0\dot{v}\\ {}^0\dot{\omega}\end{bmatrix}$$

$$=\begin{bmatrix}{}^0\dot{v}_1+{}^0\dot{\omega}_1\times P+{}^0\omega_1\times\dot{P}+{}^0R_1{}^1\dot{v}_2+{}^0\omega_1\times{}^0R_1{}^1v_2\\ {}^0\dot{\omega}_1+{}^0R_1{}^1\dot{\omega}_2+{}^0\omega_1\times{}^0R_1{}^1\omega_2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -(\dot{P}+{}^0v_2)\times\\ 0 & -{}^0\omega_2\times\end{bmatrix}\begin{bmatrix}{}^0v_1\\ {}^0\omega_1\end{bmatrix}+\begin{bmatrix}T & -P\times\\ 0 & T\end{bmatrix}\begin{bmatrix}{}^0\dot{v}_1\\ {}^0\dot{\omega}_1\end{bmatrix}$$

$$+\begin{bmatrix}{}^0\dot{v}_2\\ {}^0\dot{\omega}_2\end{bmatrix}=\dot{a}\dot{x}_1+a\ddot{x}_1+\ddot{x}_2 \tag{5}$$

where $\dot{P}={}^0\omega_1\times P+{}^0v_2$, $\dot{a}=\begin{bmatrix}0 & -(\dot{P}+{}^0v_2)\times\\ 0 & -{}^0\omega_2\times\end{bmatrix}$, $\ddot{x}_1=\begin{bmatrix}{}^0\dot{v}_1\\ {}^0\dot{\omega}_1\end{bmatrix}$, $\ddot{x}_2=\begin{bmatrix}{}^0\dot{v}_2\\ {}^0\dot{\omega}_2\end{bmatrix}$, ${}^0v_2={}^0R_1{}^1v_2$, and ${}^0\omega_2={}^0R_1{}^1\omega_2$.

2.2 Kinematic coordination and dynamic control

The dynamic control of a redundant manipulator with more than one redundant degree of freedom is not suitable for real-time implementation without an efficient computational scheme. Treating a redundant manipulator as two serially linked non-redundant manipulators reduces the computational cost considerably (Tzafestas et al., 1988 ; Horak, 1984). In Horak's algorithm (Tzafestas et al., 1988), the evaluation of velocity and acceleration at the base of the F-ARM is needed. Instead of the evaluated motion, the optimal motion can be obtained through the kinematic resolution of redundancy (Lee and Lee, 1990). As shown in (4a), $\dot{x}$ can be expressed explicitly in terms of $\dot{\theta}_b$ and $\dot{\theta}_f$. Therefore, the desired task, $\dot{x}_d$, can be distributed to each arm according to its kinematic characteristics represented by an actual manipulability ellipsoid (defined by ${}^0J_b$ and ${}^0J_f$ respectively for each arm) and the task requirements represented by a desired manipulability ellipsoid (Lee and Lee, 1988). Through this task distribution, an optimal motion at the base of the F-ARM can be determined. With the predetermined motion trajectories of the base and the end-effector of the F-ARM, the dynamic equations of the F-ARM can be directly derived by the Newton-Euler method. The dynamic equations of the B-ARM can also be derived by either the Lagrangian or Newton

-Euler method with the incorporation of the effect of the F-ARM in the form of an external force/moment at the ELBOW.

The dynamic equations of the virtually isolated local manipulators are represented in joint space as follows :

$$\tau_{b0}=A_b(\theta_b)\,\ddot{\theta}_b+B_b(\theta_b,\ \dot{\theta}_b) \tag{6a}$$
$$\tau_{f0}=A_f(\theta_f)\,\ddot{\theta}_f+B_f(\theta_f,\ \dot{\theta}_f) \tag{6b}$$

where A, B, and τ represent the inertia matrix, the Coriolis, centrifugal and gravity forces, and the generalized force in joint space, respectively, with $_b$ and $_f$ referring to the B-ARM and the F-ARM.

When the arms are serially linked together, the dynamic equations of the local arms can be derived in parallel by using each other's kinematic and dynamic characteristics. With the predetermined $\dot{x}_1$ and $\ddot{x}_1$, the torque of the F-ARM, τ_f, for the generation of local motion, $\dot{x}_2$ and $\ddot{x}_2$, can be calculated based on the classical Newton-Euler dynamic equations. Through the process of computing τ_f, the reactive force, F_R, which is transmitted to the B-ARM as the external force, can be calculated. Therefore, the torque of the B-ARM can be computed in parallel with the torque of the F-ARM as follows :

$$\tau_b=\tau_{b0}+{}^0J_1^TF_R \tag{7}$$

where F_R can be obtained by the Newton-Euler method, and ${}^0J_1$ is defined by $\dot{x}_1={}^0J_1\,\dot{\theta}_b$. Note that τ_{b0} can be calculated in parallel with F_R since τ_{b0} is the joint torque of B-ARM without the F-ARM. F_{ext} in Fig. 1 represents the external force at the HAND.

3. Dynamic Model of a Serially Linked Manipulator

Dynamic control in real-time is very difficult on account of the complex dynamics of redundant manipulators. The computational bottleneck of many advanced control schemes is the algorithm for the computation of the actuator torques which are required to produce desired joint accelerations for a given set of joint velocities and angles. The decomposition of a redundant manipulator may produce a considerable reduction in the computational cost. However, complex dynamic interactions exist between the local arms, which need to be analyzed to accomplish dynamic coordination of the local arms.

3.1 Cartesian space dynamic model

To derive the dynamic equations of a decomposed-redundant manipulator, we model each local manipulator in Cartesian space as an *equivalent kinetic energy matrix*, Λ, and an *equivalent potential energy matrix*, Λ_P, with a corresponding *Cartesian force vector*, F.

The dynamic equations of a manipulator derived in joint space, $\tau=A\ddot{\theta}+B(\theta,\ \dot{\theta})$, are converted into Cartesian space as

$$F=\Lambda\ddot{x}+V(x,\ \dot{x}) \tag{8}$$

where $\Lambda=(JA^{-1}J^T)^{-1}$ and V is the vector of end-effector centrifugal, Coriolis and gravity forces. The conversion is accomplished in keeping with the fact that the kinetic energy of the system is the same, whether it is derived in joint space or in Cartesian space (Khatib, 1987). This *effective kinetic energy matrix* represents the relation between the Cartesian acceleration and the generalized Cartesian force where the number of joints is equal to the dimension of the task space.[1] However, in handling the two local arms each of which has a number of joints less than the dimension of the task space, the effective kinetic energy matrix is not suitable. Here, we derive a more general form of the effective kinetic energy matrix, renamed as an *equivalent kinetic energy matrix*, and also derive an *equivalent potential energy matrix*, both of which can be defined regardless of the number of the joints.

Transformation of the Kinetic Energy Matrix

The equivalent kinetic energy matrix, Λ, for a p ($p<6$) degrees of freedom manipulator can be obtained through the following process.

The equivalence property of the kinetic energy in joint space and in Cartesian space is represented as

[1] More precisely speaking, when the *realizable Cartesian velocity subspace* (Lee and Lee, 1990) is equal to the task space.

$$\dot{x}^T \Lambda \dot{x} = \dot{\theta}^T A \dot{\theta} \quad (9)$$

where $\dot{x}_{m\times 1} = J_{m\times p}\dot{\theta}_{p\times 1}$, $A = J^T \Lambda J$, and $p < m = 6$. Through the SVD, $J_{m\times p}$ is represented as

$$J_{m\times p} = [U_E \;\; U_N]_{m\times n}\begin{bmatrix} S_E & 0 \\ 0 & 0 \end{bmatrix}_{m\times p} V^T_{p\times p} \quad (10)$$

where $S_E = diag[\sigma_1 \cdots \sigma_p]$.

Substituting (10) into Eq., $A = J^T \Lambda J$, we have

$$A = V S_E U_E^T \Lambda U_E S_E V^T \quad (11\text{-a})$$

$$0 = U_N^T \Lambda U_N \quad (11\text{-b})$$

Therefore, the equivalent kinetic energy matrix Λ can be defined as

$$\Lambda = \begin{bmatrix} V S_E U_E^T & 0 \\ 0 & U_N^T \end{bmatrix}^{-1} \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_E S_E V^T & 0 \\ 0 & U_N \end{bmatrix}^{-1} \quad (12)$$

Transformation of the Potential Energy Matrix

The equivalent potential energy matrix, Λ_P, for a multi-body system at the end-effector can be obtained by transforming the potential energy of each link to the end-effector :

$$U = \sum_{i=1}^{p} x_i^T H_i g = x^T \Lambda_P g \quad (13)$$

where we take $g = [0, 0, -9.8m/s^2, 0, 0, 0]^T$.

Therefore, to satisfy the above relation (13), the *equivalent potential energy matrix* at the end-effector is defined as follows :

$$\Lambda_P = \sum_{i=1}^{p} diag[x_i(1)/x(1) \;\; x_i(2)/x(2) \cdots x_i(m)/x(m)]^T \; H_i \quad (14)$$

where $x(m)$ represents the m^{th} element of the position/orientation vector, x. Note that $x(3)$ is the only essential variable such that unless $x(3) = 0$, Λ_P can be determined.

Through these equivalent energy matrices, the kinetic energy and the potential energy of each link is equivalently transformed to the end-effector of the manipulator. As a result, a manipulator is modeled as an equivalent kinetic energy matrix and an equivalent potential energy matrix at the end-effector with an ideal force source as shown in Fig. 1(b).

3.2 Dynamic equations of a decomposed-redundant manipulator

With the definition of the Cartesian dynamic model, a decomposed-redundant manipulator (refer to Fig. 1(b)) can be characterized by the equivalent kinetic energy matrices, Λ_1 and Λ_2, and the equivalent potential energy matrices, Λ_{P1} and Λ_{P2} :

Kinetic energy of B-ARM : $T_1 = 1/2\,\dot{x}_1^T \Lambda_1 \dot{x}_1$
Potential energy of B-ARM : $V_1 = x_1^T \Lambda_{P1} g$
Kinetic energy of F-ARM : $T_2 = 1/2\,\dot{x}^T \Lambda_2 \dot{x}$
Potential energy of F-ARM : $V_2 = x^T \Lambda_{P2} g$.

Note that the F-ARM generates the motion $\dot{x}_2$ and $\ddot{x}_2$ with respect to its base (ELBOW) and that the actual motion at the end-effector of F-ARM (HAND), $\dot{x}$ and $\ddot{x}$, is the aggregation of the motions of both the B-ARM and the F-ARM, as shown in Fig. 1(a). Note also that two independent parameters, $\dot{x}_1$ and $\dot{x}_2$, form a complete set of velocity parameters of the decomposed-redundant manipulator in Cartesian space and thus constitute a system of generalized coordinates for the decomposed-redundant manipulator.

To derive the dynamic equations of the decomposed-redundant manipulator, we define the *Lagrangian function* L as :

$$L = T - V = 1/2\,\dot{x}_1^T \Lambda_1 \dot{x}_1 + 1/2\,\dot{x}^T \Lambda_2 \dot{x} - x_1^T \Lambda_{P1} g - x^T \Lambda_{P2} g \quad (15)$$

where T and V are the manipulator kinetic energy and potential energy, respectively.

The Lagrangian equations of motion for this decomposed-redundant manipulator are

$$F_1 = d/dt\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = d/dt\left(\Lambda_1 \dot{x}_1 + \left(\frac{\partial \dot{x}}{\partial \dot{x}_1}\right)^T \Lambda_2 \dot{x}\right) - \frac{\partial (x_1^T \Lambda_{P1} + x^T \Lambda_{P2})}{\partial x_1} g - \frac{\partial KE}{\partial x_1} \quad (16a)$$

$$F_2 = d/dt\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = d/dt(\Lambda_2 \dot{x}_1) - \frac{\partial (x^T \Lambda_{P2})}{\partial x_2} g - \frac{\partial KE_2}{\partial x_2} \quad (16b)$$

where $KE = 1/2\,\dot{x}_1^T \Lambda_1 \dot{x}_1 + 1/2\,\dot{x}^T \Lambda_2 \dot{x}$ and $KE_2 =$

$1/2\dot{x}^T\Lambda_2\dot{x}$.

By using the kinematic relations among $\dot{x}$, $\dot{x}_1$, and $\dot{x}_2$,

$$\dot{x}=a\dot{x}_1+\dot{x}_2$$
$$\partial\dot{x}/\partial\dot{x}_1=a \text{ and } \partial\dot{x}/\partial\dot{x}_2=\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (17)$$

(16a) and (16b) can be rewritten as

$$F_1=d/dt(\Lambda_1\dot{x}_1+a^T\Lambda_2\dot{x}) -\frac{\partial(x_1^T\Lambda_{P1}+x^T\Lambda_{P2})}{\partial x_1}g-\partial KE/\partial x_1 \quad (18a)$$

$$F_2=d/dt(\Lambda_2\dot{x})-\frac{\partial(x^T\Lambda_{P2})}{\partial x_2}g -\partial KE_2/\partial x_2 \quad (18b)$$

Taking time derivatives of the right sides of (18a) and (18b), we have

$$F_1=a^T\Lambda_2\ddot{x}+\Lambda_1\ddot{x}_1+\dot{\Lambda}_1\dot{x}_1+a^T\dot{\Lambda}_2\dot{x} +\dot{a}^T\Lambda_2\dot{x}-\frac{\partial(x_1^T\Lambda_{P1}+x^T\Lambda_{P2})}{\partial x_1}g -\partial KE/\partial x_1 \quad (19a)$$

$$F_2=\Lambda_2\ddot{x}+\dot{\Lambda}_2\dot{x}-\frac{\partial(x^T\Lambda_{P2})}{\partial x_2}g -\partial KE_2/\partial x_2 \quad (19b)$$

Using the relation $\ddot{x}=a\ddot{x}_1+\ddot{x}_2+\dot{a}\dot{x}_1$, which is shown in Section 2, (19a) and (19b) are combined in matrix form :

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}=\begin{bmatrix} a^T\Lambda_2a+\Lambda_1 & a^T\Lambda_2 \\ \Lambda_2a & \Lambda_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} F_{nr1} \\ F_{nr2} \end{bmatrix}. \quad (20)$$

where $F_{nr1}=a^T\Lambda_2\dot{a}\dot{x}_1+\dot{\Lambda}_1\dot{x}_1+(a^T\dot{\Lambda}_2+\dot{a}^T\Lambda_2)\dot{x}-\frac{\partial(x_1^T\Lambda_{P1}+x^T\Lambda_{P2})}{\partial x_1}g-\partial KE/\partial x_1$, and $F_{nr2}=\Lambda_2\dot{a}\dot{x}_1+\dot{\Lambda}_2\dot{x}-\frac{\partial(x^T\Lambda_{P2})}{\partial x_2}g-\partial KE_2/\partial x_2$.

The corresponding joint torques for each manipulator can be calculated as :

$$\tau_b={}^0J_1^TF_1 \quad (21a)$$
$$\tau_f={}^0J_f^TF_2 \quad (21b)$$

It can be seen from (20) and (21) that each arm's joint torques for the current motion are not only dependent upon its own motion, but also upon the other arm's motion. This dynamic interaction between the local arms is clearly represented by the dynamic equations based on the Cartesian space dynamic model. When there is an external contacting force at the HAND of the manipulator, $-F_{ext}$, extra joint torques are required to resist the static force for each arm, and the total joint torque for each arm is given as :

$$\tau_b={}^0J_1^T(F_1+\Gamma^{-1}F_{ext}) \quad (22a)$$
$$\tau_f={}^0J_f^T(F_2+F_{ext}) \quad (22b)$$

where $\Gamma=\begin{bmatrix} I & 0 \\ P & I \end{bmatrix}$.

4. Dynamic Resolution of Redundancy

Here, we will introduce a new concept of the task (a desired acceleration) distribution to each local arm according to its dynamic characteristics which are abstracted by the Cartesian Force Ellipsoid (C. F. E) and Cartesian Acceleration Ellipsoid (C. A. E).

4.1 Capability of generating cartesian force

The limited joint torque range can be mapped into a feasible range of Cartesian force. From the *Principle of Virtual Work*, we have the relation between the joint torque, τ, and the Cartesian force, F, i. e., $\tau=J^TF$. The inverse mapping of this relation, $\tau \xrightarrow{(J^T)^+} F$[2], gives the Cartesian force range which can be generated by the limited joint torque range.

The joint torque ranges of each manipulator, the B-ARM and the F-ARM, are represented as

$$\tau_{b,min}\leq\tau_b\leq\tau_{b,max},\ \tau_{f,min}\leq\tau_f\leq\tau_{f,max}.$$

Using the dynamic Eq. (20) with the current joint velocities and angles, new joint torque ranges which can be used to generate the Cartesian acceleration are obtained :

$$\tau_{b,min}-{}^0J_1^TF_{nr1}\leq\tilde{\tau}_b\leq\tau_{b,max}-{}^0J_1^TF_{nr1},$$
$$\tau_{f,min}-{}^0J_1^TF_{nr2}\leq\tilde{\tau}_f\leq\tau_{f,max}-{}^0J_f^TF_{nr2}.$$

To obtain the feasible force range through the inverse mapping, a normalized torque, τ^*, is defined and denoted by

$$\tau^*=W\tilde{\tau} \quad (23)$$

where a weighting matrix, $W=diag[1/\tilde{\tau}_{i,limit}]\in R^{n\times n}$ with $\tilde{\tau}_{i,limit}=min(|\tilde{\tau}_{i,max}|, |\tilde{\tau}_{i,min}|)$. The

[2] $(J^T)^+$ is the generalized inverse of J^T.

normalized torque range (hyper-cube) can be approximated as a hyper-sphere (refer to Fig. 2) such that the allowable torque range is represented as $\|\tau^*\| \le 1$. Now, the corresponding Cartesian force range through the mapping, $\tau^* \xrightarrow{((JW^T)^T)^{-}} F^*$, can be represented as a Cartesian Force Ellipsoid :

Definition : Cartesian Force Ellipsoid (C. F. E)
A hyper-ellipsoid which represents the feasible range of Cartesian force within the allowable torque range, i. e., the *capability of generating Cartesian force*.

$$\|\tau^*\|^2 = \tau^{*T}\tau^*$$
$$= (W\tilde{\tau})^T(W\tilde{\tau}) \; (\longleftarrow \; \tilde{\tau} = J^T F^*)$$
$$= F^{*T} J W^T (J W^T)^T F^* \le 1 \qquad (24)$$

Definition : Realizable Cartesian Force Subspace derived from JW^T, $R_F(JW^T)$

$R_F(JW^T) \subseteq R(F^*)$ such that $\forall F^* \in R_F(JW^T)$, $\exists \tau^* | \tau^* = (JW^T)^T F^*$

Definition : Unrealizable Cartesian Force Subspace derived from JW^T, $R_F^{\perp}(JW^T)$

$R_F^{\perp}(JW^T) \subseteq R(F^*)$ such that $\forall F^* \in R_F(JW^T)$, $\nexists \tau^* | \tau^* = (JW^T)^T F^*$

Furthermore, $R_F(JW^T) \cap R_F^{\perp}(JW^T) = \phi$ and $R_F(JW^T) + R_F^{\perp}(JW^T) = R(F^*)$.

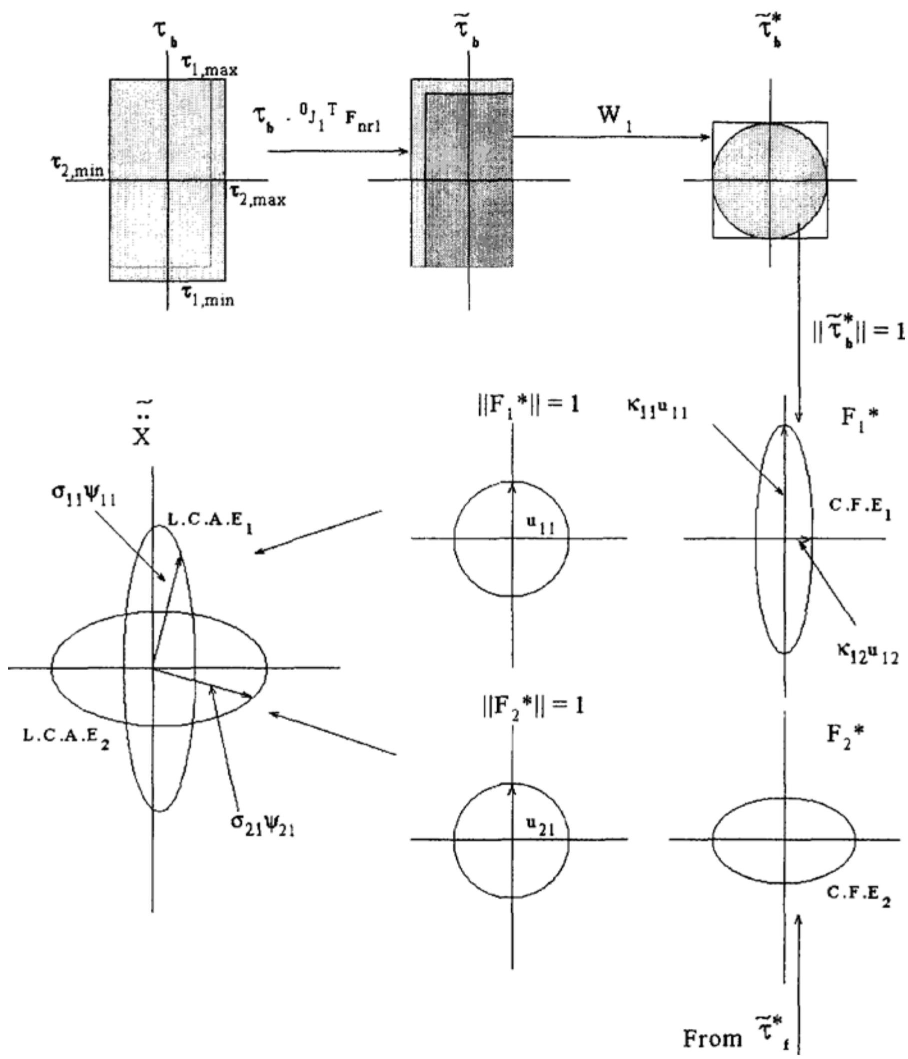


Fig. 2 Task distribution through C.F.E and L.C.A.E

With the Jacobian matrices and weighting matrices of each arm (${}^0J_1$ and W_1 for the B-ARM, ${}^0J_f$ and W_2 for the F-ARM), the C. F. E for the B-ARM, *C. F. E*$_1$, and the C. F. E for the F-ARM, *C. F. E*$_2$, are defined as follows :

$$C.F.E_1 : F_1^{*T}U_1K_1^2U_1^TF_1^* \leq 1 \tag{25a}$$

where $K_1 = diag[x_{11}\ x_{12}\ \cdots\ x_{1b}]$ (the inverse of non-zero singular values of ${}^0J_fW_2^T$) and $U_2 = [u_{21}\ u_{22}\ \cdots\ u_{2f}]$ (the left-singular vectors corresponding to K_2).
Note that the coeffcient matrix of the C.F.E, $JW^T(JW^T)^T$, can be simply repressnted by $U_2K_2U_S^T$, where $K_2 = diag[x_1^2\ x_2^2\ \cdots\ x_p^2]$ (Lee and Lee, 1990).

4.2 Efficiency of generating cartesian acceleration

The dynamic equations of the decomposed-redundant manipulator, (20), represent the required Cartesian forces, F_1 and F_2, for the current acceleration, $\ddot{x}_1$ and $\ddot{x}_2$. Consider now how the desired acceleration, $\ddot{x}_d$, can be generated by the Cartesian forces, F_1^* and F_2^* residing in the realizable Cartesian force subspaces. Since the acceleration at the hand is represented as $\ddot{x} = \alpha\ddot{x}_1 + \ddot{x}_2 + \dot{\alpha}\dot{x}_1$, the desired acceleration, $\ddot{x}_d$ can be achieved by the accelerations of local manipulators,

$$\tilde{\ddot{x}}_d = \alpha\ddot{x}_1 + \ddot{x}_2 \tag{26}$$

where $\tilde{\ddot{x}} = \ddot{x}_d - \dot{\alpha}\dot{x}_1$.

Note that the acceleration can be generated only by the force which resides in the realizable Cartesian force subspace. Therefore, the accelerations, $\ddot{x}_1$ and $\ddot{x}_2$, need to be described in terms of Cartesian forces F_1^* and F_2^* using (20).

Definition : Interaction Cartesian Force Subspace, R_F^{IN}

The Cartesian force subspace in which interactions between two local arms exist such that $R_F^{IN} = R_F({}^0J_1W_1^T) \cap R_F(\Gamma^{-1}{}^0J_fW_2^T)$.

Lemma 1. The set of basis vectors ${}^{IN}U$ of $R_F^{IN} = R_F({}^0J_1W_1^T) \cap R_F(\Gamma^{-1}{}^0J_fW_2^T)$, can be obtained by ${}^{IN}U = [{}^1U^\perp, {}^fU^\perp]^\perp$, where ${}^1U$ and ${}^fU$ are respectively the set of basis vectors of $R_F({}^0J_1W_1^T)$ and $R_F(\Gamma^{-1}{}^0J_fW_2^T)$, and $U^\perp$ represents the set of basis vectors for the orthogonal complement of the subspace spanned by U.

Proof : Since $R_F(\Gamma^{-1}{}^0J_fW_2^T) = [R_F(\Gamma^{-1}{}^0J_fW_2^T)^\perp]^\perp$, we have
$R_F({}^0J_1W_1^T) \cap R_F(\Gamma^{-1}{}^0J_fW_2^T) = [R_F({}^0J_1W_1^T)^\perp]^\perp \cap [R_F(\Gamma^{-1}{}^0J_fW_2^T)^\perp]^\perp = [R_F({}^0J_1W_1^T)^\perp + R_F(\Gamma^{-1}{}^0J_fW_2^T)^\perp]^\perp$
where $\perp$ represents the orthogonal complement of the corresponding subspace.
$R_F({}^0J_1W_1^T)$ and $R_F(\Gamma^{-1}{}^0J_fW_2^T)$ are spanned respectively by ${}^1U$ and ${}^fU$, and, consequently, $R_F({}^0J_1W_1^T)^\perp + R_F(\Gamma^{-1}{}^0J_fW_2^T)^\perp$ is spanned by $[{}^1U^\perp, {}^fU^\perp]$. Therefore, $[R_F({}^0J_1W_1^T)^\perp + R_F(\Gamma^{-1}{}^0J_fW_2^T)^\perp]^\perp$ is spanned by $[{}^1U^\perp, {}^fU^\perp]^\perp$. (Q.E.D)

Note that only the force component which resides in the interaction Cartesian force subspace contributes to the dynamic interaction between the local arms. Therefore, before multiplying the inverse of $\begin{bmatrix} \alpha^T\Lambda_2\alpha + \Lambda_1 & \alpha^T\Lambda_2 \\ \Lambda_2\alpha & \Lambda_2 \end{bmatrix}$ to both sides of (20) to represent the accelerations in terms of Cartesian forces, $\alpha^T\Lambda_2\ddot{x}_2$, the F_1 component representing the interaction force, is projected onto the interaction subspace, R_F^{IN} ; likewise $\Lambda_2\alpha\ddot{x}_1$, the F_2 component representing the interaction force, is transformed to the elbow and projected onto the interaction subspace, and is transformed back to the hand.

Consequently, the accelerations which can be generated by the forces residing in the realizable Cartesian force subspaces are represented by

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} F_1^* \\ F_2^* \end{bmatrix} \tag{27}$$

where $F_1^* = F_1 - F_{nr1}$, $F_2^* = F_2 - F_{nr2}$, $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \alpha^T\Lambda_2\alpha + \Lambda_1 & P_{IN}\alpha^T\Lambda_2 \\ \Gamma P_{IN}\Gamma^{-1}\Lambda_2\alpha & \Lambda_2 \end{bmatrix}^{-1}$, and the projection operator $P_{IN} = {}^{IN}U({}^{IN}U^T\ {}^{IN}U)^{-1}{}^{IN}U^T$.

Equation (27) is rewritten to represent the Cartesian acceleration generated by the B-ARM and by the F-ARM individually :

$$\ddot{x}_1 = [h_{11}\ h_{12}] \begin{bmatrix} F_1^* \\ F_2^* \end{bmatrix} \tag{28a}$$

$$\ddot{x}_2 = [h_{21}\ h_{22}] \begin{bmatrix} F_1^* \\ F_2^* \end{bmatrix} \tag{28b}$$

Substituting (28a) and (28b) into $\ddot{x}_d = a\ddot{x}_1 + \ddot{x}_2$,

$$\ddot{x}_d = [ah_{11} + h_{21} \;\; ah_{12} + h_{22}] \begin{bmatrix} F_1^* \\ F_2^* \end{bmatrix}. \qquad (29)$$

Eq. (29) represents the Cartesian acceleration at the hand generated by F_1^* and F_2^* which are the Cartesian forces to be used for the generation of Cartesian acceleration within the allowable torque range.

Generally, the local arms do not have enough degrees of freedom to cover the task space. The following relationships exist between the rank of ${}^0J_1 W_1^T$ and ${}^0J_f W_2^T$ and the dimension of the task space :

$$r({}^0J_1 W_1^T),\; r({}^0J_f W_2^T) \leq m \text{ and}$$
$$r({}^0J_1 W_1^T) + r({}^0J_f W_2^T) = n > m \qquad (30a)$$

where the local arms are assumed to be non-singular, m is the dimension of task space and n is the number of joints in the decomposed-redundant manipulator.

The dimension of the interaction Cartesian force subspace can also be obtained by

$$dim(R_F^{IN}) = dim(R_F({}^0J_1 W_1^T)) + dim(R_F({}^0J_f W_2^T)) - m \qquad (30b)$$

where $r({}^0J_f W_2^T)$ is the same as $r(\Gamma^{-1} {}^0J_f W_2^T)$ by *Sylvester's inequality* since Γ^{-1} has full rank.

Note that $dim\ (R_F^{IN}) \geq n - m$ unless the local arms are singular, and that $dim(R_F^{IN}) > n - m$ when the decomposed-redundant manipulator is singular. Eq. (30) implies that among the basis vectors of $R_F({}^0J_1 W_1^T)$, only the ones which do not reside in the interaction Cartesian force subspace, R_F^{IN}, directly contribute to the acceleration at the hand.

Theorem 1 : F_1^*, the force supplied by the B-ARM and residing in R_F^{IN} cannot generate any acceleration by itself at the hand.

Proof : The acceleration generated by the B-ARM results in acceleration at the base of the F-ARM. When F_1^* resides in R_F^{IN}, the positive acceleration of the elbow is absorbed as the negative acceleration of the F-ARM if the F-ARM does not provide the opposite force to resist it. Therefore, F_1^* by itself does not contribute to the acceleration at the hand. (Q. E. D)

Each arm's direct efficiency of generating Cartesian acceleration can be respectively defined through the mapping, $F_1^* \xrightarrow{ah_{11}+h_{21}} \ddot{x}$ and $F_2^* \xrightarrow{ah_{12}+h_{22}} \ddot{x}$ (refer to Eq. (29)).

Definition : Direct Cartesian Acceleration Ellipsoid (DCAE)

A hyper-ellipsoid which represents the feasible acceleration range at the hand corresponding to the unit norm Cartesian force which resides in the realizable Cartesian force subspace, i. e., the *direct efficiency of generating Cartesian acceleration.*

The DCAE for the B-ARM, *D. C. A. E*$_1$, and for the F-ARM, *D. C. A. E*$_2$, are defined as

$$D.C.A.E_1 : \ddot{x}^T C_1 \ddot{x} - 1 = 0$$
$$\text{where } C_1 = \{(ah_{11} + h_{21})(ah_{11} + h_{21})^T\}^{-1}$$
$$D.C.A.E_2 : \ddot{x}^T C_2 \ddot{x} - 1 = 0$$
$$\text{where } C_2 = \{(ah_{12} + h_{22})(ah_{12} + h_{22})^T\}^{-1}.$$

Since the unit force vector on the C. F. E is uniquely mapped onto a vector on the D. C. A. E, a *feasible acceleration vector* on the D. C. A. E is defined as a vector mapped from a principal axis of C. F. E, i. e., u_{1i} of *C. F. E*$_1$ and u_{2i} of *C. F. E*$_2$.

Definition : Direct Feasible Acceleration Vectors, $\sigma_{ji}\phi_{ji}$

Direct Feasible Acceleration Vectors of B-ARM : $\sigma_{1i}\phi_{1i} = (ah_{11} + h_{21}) u_{1i}$, $i = 1$ to $(m - f)$

Direct Feasible Acceleration Vectors of F-ARM : $\sigma_{2i}\phi_{2i} = (ah_{12} + h_{22}) u_{2i}$, $i = 1$ to f.

Definition : Direct Feasible Acceleration Vector Sets, $\Sigma_1 \Psi_1$ and $\Sigma_2 \Psi_2$,

where $\Sigma_1 = diag[\sigma_{11} \; \sigma_{12} \cdots \sigma_{1(m-f)}]$,

$\Sigma_2 = diag[\sigma_{21} \; \sigma_{22} \cdots \sigma_{2f}]$,

$\Psi_1 = [\phi_{11} \; \phi_{12} \cdots \phi_{1(m-f)}]$ and $\Psi_2 = [\phi_{21} \; \phi_{22} \cdots \phi_{2f}]$.

The desired acceleration $\ddot{x}_d$ can be achieved by using Ψ_1 and Ψ_2 since *rank*$[\Psi_1, \Psi_2]$ is equal to *rank*$[\Psi_1, \Psi_2, \ddot{x}_d]$ unless $dim(R_F^{IF}) > n - m$.

4.3 Hierarchical task distribution

The highest priority of the F-ARM is effective for an instant and provides a unique solution for the task distribution. However, it may drive the F-ARM to a singular configuration by heavy loads, since it has a relatively small workspace.

Further, the joint torques are functions of the joint velocities and manipulator configurations, which are highly nonlinear, so that if the task distribution is done solely based on the direct efficiency of generating Cartesian acceleration, it might be the case that excessive joint torques are required for the F-ARM.

4.3.1 Local efficiency of generating cartesian acceleration

As shown in (27), h_{11} and h_{22} represent how efficiently the local accelerations, $\ddot{x}_1$ and $\ddot{x}_2$, are generated respectively by F_1^* and F_2^*. Therefore, each arm's efficiency in generating Cartesian acceleration at the hand can be respectively defined throughthe mapping, $F_1^* \xrightarrow{ah_{11}} \tilde{\ddot{x}}$ and $F_2^* \xrightarrow{h_{22}} \tilde{\ddot{x}}$.

Definition : Local Cartesian Acceleration Ellipsoid (LCAE)

A hyper-ellipsoid which represents the feasible acceleration range at the hand corresponding to the unit norm Cartesian force, provided by each local arm without considering the interactions, i. e., the *local efficiency of generating Cartesian acceleration.*

The LCAE for the B-ARM, $L.C.A.E_1$, and for the F-ARM, $L.C.A.E_2$, are defined as

$$L.C.A.E_1 : \tilde{\ddot{x}}^T C_1 \tilde{\ddot{x}} - 1 = 0$$

$$\text{where } C_1 = (ah_{11}h_{11}^T a^T)^{-1}$$

$$L.C.A.E_2 : \tilde{\ddot{x}}^T C_2 \tilde{\ddot{x}} - 1 = 0$$

$$\text{where } C_2 = (h_{22}h_{22}^T)^{-1}$$

Definition : Local Feasible Acceleration Vectors, $\sigma_{ji}\psi_{ji}$

Local Feasible Acceleration Vectors of B-ARM : $\sigma_{1i}\psi_{1i} = ah_{11}u_{1i}$, $i=1$ to b

Local Feasible Acceleration Vectors of F-ARM : $\sigma_{2i}\psi_{2i} = h_{22}u_{2i}$, $i=1$ to f.

Definition : Local Feasible Acceleration Vector Sets, $\Sigma_1\Psi_1$ and $\Sigma_2\Psi_2$,

where $\Sigma_1 = diag[\sigma_{11}\sigma_{12}\cdots\sigma_{1b}]$, $\Sigma_2 = diag[\sigma_{21}\sigma_{22}\cdots\sigma_{2f}]$,

$\Psi_1 = [\psi_{11}\psi_{12}\cdots\psi_{1b}]$ and $\Psi_2 = [\psi_{21}\psi_{22}\cdots\psi_{2f}]$.

4.3.2 Dynamic task distribution

The task represented by $\tilde{\ddot{x}}_d$ can now be optimally distributed to each local feasible acceleration vector, ψ_{ji}, $j=1$ or 2, according to the capability and efficiency of the manipulator in generating ψ_{ji}. The interaction effects, $h_{12}F_2^*$ on $\ddot{x}_1$ and $h_{21}F_1^*$ on $\ddot{x}_2$, with other global requirements can be incorporated into the task distribution by the factor, γ, which governs the task-sharing between the local arms.

Task Distribution Criteria

C1 : The task is decomposed into individual (unit) local feasible acceleration vectors proportional to their $\sigma_{ji}\chi_{ji}$ values, where χ_{ji} represents the capability of generating Cartesian force, u_{ji}, and σ_{ji} represents the local efficiency of generating Cartesian acceleration, ψ_{ji}, by u_{ji}.

C2 : A *task distribution factor*, γ, is defined so that the task-sharing between two local arms is adjusted to the global task requirements, for example, joint torque limit avoidance and singularity avoidance.

The task distribution can be summarized as the minimization of

$$Cost = \frac{(1-\gamma)}{2}\sum_{i=1}^{b}\left(\frac{k_{1i}}{\sigma_{1i}\cdot\chi_{1i}}\right)^2 + \frac{\gamma}{2}\sum_{i=1}^{f}\left(\frac{k_{2i}}{\sigma_{2i}\cdot\chi_{2i}}\right)^2 \tag{31}$$

subject to

$$\tilde{\ddot{x}}_d = \Psi_1 K_1 + \Psi_2 K_2 \tag{32}$$

where $0 \le \gamma \le 1$, $K_1 = [k_{11}k_{12}\cdots k_{1b}]^T$, and $K_2 = [k_{21}k_{22}\cdots k_{2f}]^T$.

This task distribution procedure is illustrated in Fig. 2.

Calculation of K_1, and K_2

From (32), we have

$$\tilde{\ddot{x}}_d = \Psi K \tag{33}$$

where $\tilde{\ddot{x}}_d \in R^{m\times 1}$, $\Psi = [\Psi_1 : \Psi_2] \in R^{m\times(b+f)}$, and $K = [K_1^T : K_2^T] \in R^{(b+f)\times 1}$.

The performance function, (31) can be reformulated as

$$G = 1/2[K_1^T \ K_2^T]\, diag[w_1\cdots w_b \ w_{b+1}\cdots w_{b+f}]\begin{bmatrix}K_1\\K_2\end{bmatrix} = 1/2K^T W K \tag{34}$$

where $w_i = \dfrac{1-\gamma}{(\sigma_{1i}\cdot\chi_{1i})^2}$ for $i=1, \cdots, b$; $w_{b+i} = \dfrac{\gamma}{(\sigma_{2i}\cdot\chi_{2i})^2}$ for $i=b+1, \cdots, b+f$.

To obtain K such that $min\ G(K)$ subject to

$\ddot{\tilde{x}}_d = \Psi K$, let us define the Lagrangian $L(K)$ as follows (Chang, 1986) :

$$L(K) = \lambda^T F(K) + G(K) \tag{35}$$

where $F(K) = \ddot{\tilde{x}}_d - \Psi K$ and λ is an $m \times 1$ Lagrangian multiplier. The condition for the extremum, $\partial L/\partial K = 0$, results in

$$\Psi^T \lambda = W\ K. \tag{36}$$

Assuming that we can select m linearly independent rows from Ψ^T, (36) can be divided into

$$\Psi_m^T \lambda = W_m K \tag{37a}$$
$$\bar{\Psi}_m^T \lambda = \bar{W}_m K \tag{37b}$$

where $\Psi_m^T \in R^{m\times m}$ and $\bar{\Psi}_m^T \in R^{(b+f-m)\times m}$ are formed respectively by m linearly independent rows of Ψ^T and by the remaining $(b+f-m)$ rows of W.

Determining λ from (37a) and plugging into (37b), we have

$$\bar{\Psi}_m^T (\Psi_m^T)^{-1} W_m K = \bar{W}_m K \tag{38}$$

Now, K can be uniquely determined by (33) and (38).

5. Elbow Control of a Decomposed —Redundant Manipulator

Task distribution is not feasible when the principal axes of LCAE's (the feasible acceleration vectors) cannot cover the whole task space. This is the case when the decomposed -redundant manipulator becomes singular, i. e., $dim(R_F^{IN}) > n - m$. This might be avoided by adding some acceleration $\ddot{x}_{elbow}$ to $\Psi_1 K_1$ and $-\ddot{x}_{elbow}$ to $\Psi_2 K_2$ without affecting the Cartesian acceleration $\ddot{\tilde{x}}\ (= \Psi_1 K_1 + \Psi_2 K_2)$ at the hand, as it is well described for kinematic control of the elbow (Lee and Lee, 1990). However, this dynamic control of the elbow cannot guarantee the avoidance of kinematic singularities. To resolve this situation globally, kinematic control of the elbow can be done using the intersection subspace of the realizable Cartesian velocity subspace of the B-ARM, $R_{\dot{X}}({}^0J_b)$,[3] and the realizable Cartesian velocity subspace of the F-ARM, $R_{\dot{X}}({}^0J_f)$[4] at the hand (Lee and Lee, 1990). That is, if the B-ARM generates a certain motion V_{hand} at the hand, then the F-ARM generates $-V_{hand}$ to compensate athe B-ARM motion such that the velocity at the hand is not affected by the elbow control.

Two objectives can be considered in the kinematic control of the elbow to achieve efficient dynamic task distribution :

1. To maximize the dimension of C. F. E's.
2. To make the shape of LCAE's as round as possible to make the manipulator versatile and flexible for various tasks.

Here, we place emphasis on the first objective. In fact, making the shape of LCAE as round as possible in the workspace has been discussed as a design criterion for robotic manipulators.

Note that since the weighting matrix W is a positive-diagonal matrix, the realizable Cartesian force subspace, $R_F(JW^T)$, is the same as the the realizable Cartesian velocity subspace, $R_{\dot{X}}(J)$, and also the dimension of the C. F. E defined by JW^T is the same as the dimension of manipulability ellipsoid defined by J. Therefore, the kinematic performance function TOMM (Lee and Lee, 1988), which represents the suitability of manipulator configuration for a given task in terms of the efficiency of motion generation and the resistivity of static force, is suitable for elbow control. This elbow control is well described analytically in (Lee and Lee, 1990).

This kinematic control of the elbow can be done in parallel with the task execution when the desired acceleration is zero, i. e., at the constant velocity phase, where the dynamic characteristics are not dominant factors for the optimal task execution. In addition to the performance improvement for the task at the hand, elbow control can be also utilized for a certain subtask specified at the elbow such as avoiding obstacles

---

[3] From (4a) and (4c), $a\dot{x}_1 = {}^0J_b\dot{\theta}_b$ and $R_{\dot{X}}({}^0J_b)$ is defined as follows :
$R_{\dot{X}}({}^0J_b) \subseteq R(\dot{x})$ such that $\forall \dot{x} \in R_{\dot{X}}({}^0J_b)$, $\exists\ \dot{\theta}_b | \dot{x} = {}^0J_b\dot{\theta}_b$

[4] From (4a) and (4c), $\dot{x}_2 = {}^0J_f\dot{\theta}_f$ and $R_{\dot{X}}({}^0J_f)$ is defined as follows :
$R_{\dot{X}}({}^0J_f) \subseteq R(\dot{x})$ such that $\forall \dot{x} \in R_{\dot{X}}({}^0J_f)$, $\exists\ \dot{\theta}_f | \dot{x} = {}^0J_f\dot{\theta}_f$

during the task execution.

6. Simulation

We select a 4-revolute-joint redundant manipulator with a planar workspace. The elbow is predetermined at the end of the second link. To show the effectiveness of the task distribution scheme developed in this paper, we display the configuration of the manipulator with the needed torque of each joint. The joint torque values are displayed to show the effect of the task distribution and elbow control. For simplicity, F_{nr1} and F_{nr2} are assumed to be negligible.

As discussed in Sec. 4, the task distribution factor γ can be adjusted to avoid possible joint torque limits of each manipulator which cannot be avoided by the optimal task distribution with a fixed γ. To show the effect, we demonstrate two different cases of task executions : $\gamma=1$ (intending that the desired acceleration is generated by only the B-ARM), and $\gamma=0.1$ (intending that the desired acceleration is generated mostly by the F-ARM).

The workspace of the F-ARM is small such that there exisls the possiblity that the F-ARM is confronted with a boundary singularity when it is the primary arm used for the task execution (for example, $\gamma=0.1$). This situation is indicated by the value of TOMM. To avoid this situation globally, we use elbow control in the beginning of each task segment to make $TOMM<0.3$.

Simulation Environment

1. A redundant manipulator which has 4 links ($l_1=l_2=1.2m$ and $l_3=l_4=0.6m$) is selected and each joint-torque-limit is given as $\tau_{1,min}=-100$ $N\cdot m$, $\tau_{2,min}=-100N\cdot m$, $\tau_{3,min}=-30N\cdot m$, $\tau_{4,min}=-30N\cdot m$, $\tau_{1,max}=100N\cdot m$, $\tau_{2,max}=100$ $N\cdot m$, $\tau_{3,max}=30N\cdot m$, and $\tau_{4,max}=30N\cdot m$.

2. All the mass of each link is assumed to be located at the distal end of each link as a cubic shape such that the inertia matrix $I_i=I$. The mass values are given as $m_1=10Kg$, $m_2=5Kg$, $m_3=2.5Kg$ and $m_4=1.2Kg$.

3. The given task is composed of two task segments, where a task segment is defined as a basic unit of task which requires the same motion and static force directions :

TS_1 : Move From (1.0, 0.0) To (1.5, 0.0) with an acceleration of

$$\ddot{x}_d=\begin{bmatrix}10\\0\end{bmatrix}m/sec^2 \quad \text{while} \quad \text{x-position} \le 1.25,$$

and $\ddot{x}_d=\begin{bmatrix}-10\\0\end{bmatrix}m/sec^2$ otherwise.

TS_2 : Move From (1.5, 0.0) To (2.0, −0.1) with an acceleration of

$$\ddot{x}_d=\begin{bmatrix}50/\sqrt{26}\\-10/\sqrt{26}\end{bmatrix}m/sec^2 \quad \text{while} \quad \text{x-position} \le 1.75$$

and $\ddot{x}_d=\begin{bmatrix}-50/\sqrt{26}\\10/\sqrt{26}\end{bmatrix}m/sec^2$ otherwise.

4. A static force of $10N$ is required in the direction normal to the direction of motion for each task segment.

5. Initial location of the elbow is given as (0.5, 0.2).

6. Considering the motion and static force requirements of the task, the desired manipulability ellipsoid is defined as the principal axes and length along the axes, $\{\chi_{1d}\ {}^d u_1, \chi_{2d}\ {}^d u_2\}$, and assigned to the F-ARM (Lee and Lee, 1988). The values are given as follows :

$\chi_{1d}=1.0$, and $\chi_{2d}=0.4$ always,

${}^d u_1=[1.0\ 0]^T$, and ${}^d u_2=[0\ 1.0]^T$ for the task segment 1,

${}^d u_1=[5/\sqrt{26}\ -1/\sqrt{26}]^T$, and ${}^d u_2=[1/\sqrt{26}\ 5/\sqrt{26}]^T$ for the task segment 2.

${}^d u_1$ is given as the desired motion direction, and ${}^d u_2$ is given as the static force direction.

The desired manipulability ellipsoid is provided to guide elbow control by TOMM, which is defined as the discrepancy between the desired manipulability ellipsoid and an actual manipulability ellipsoid.

Control Scheme

1. The desired acceleration, $\tilde{\ddot{x}}_d(=\ddot{x}_d-\alpha\dot{x}_1)$ is distributed to the B-ARM and the F-ARM based on the task distribution criteria with a fixed γ (refer to (31) and (32)).

2. To see the effect of changing γ (changing the task share of each arm) on avoiding the joint torque limits, we first demonstrate two task execu-

tion examples with different values of γ.

3. To follow global task requirements, such as avoiding singularites and minimizing power consumption, the elbow control (to reshape and reorient the manipulability ellipsoid) is done in the beginning of each task segment until *TOMM* <0.3. Elbow control is demonstrated for only the case of task execution (with $\gamma=0.1$) in which the F-ARM performs the majority of the task.

Results of Simulation

1. The task is suitably distributed to each arm, as can be seen by comparing the torque value for each joint and its limit.

2. As shown by Figs. 3 and 4 by reducing the load to the B-ARM (decreasing γ), the situation of the B-ARM reaching joint torque limits-even though the load is well distributed to each joint-is resolved.

3. As it is demonstrated by Fig. 4, the amplitudes of joint torques are rapidly increasing when the configuration of the F-ARM becomes close to singular. There might be two reasons for the excessive joint torques : (a) the F-ARM has low

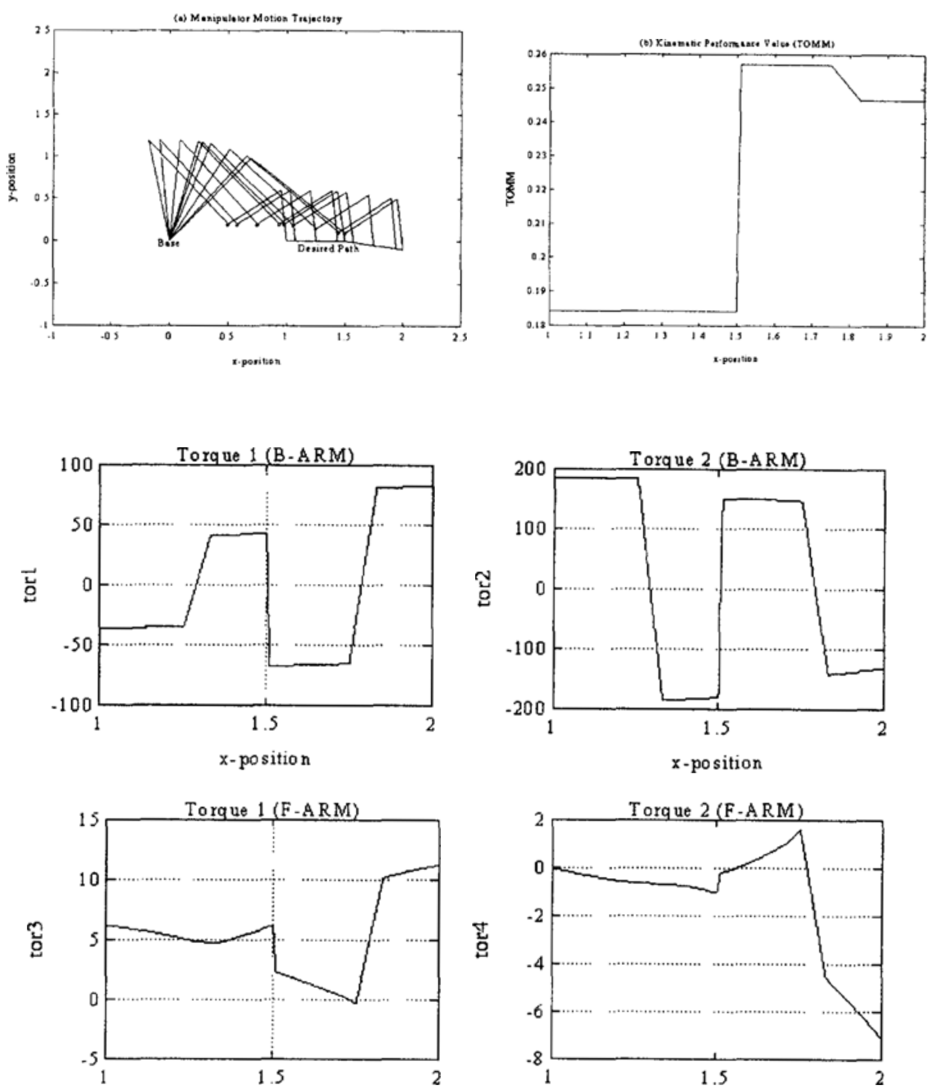


Fig. 3 Joint torques variation, $\gamma=1$,
Torque limits : $-100N \cdot m \leq \tau_1,\ \tau_2 \leq 100N \cdot m,\ -30N \cdot m \leq \tau_3,\ \tau_4 \leq 30N \cdot m$

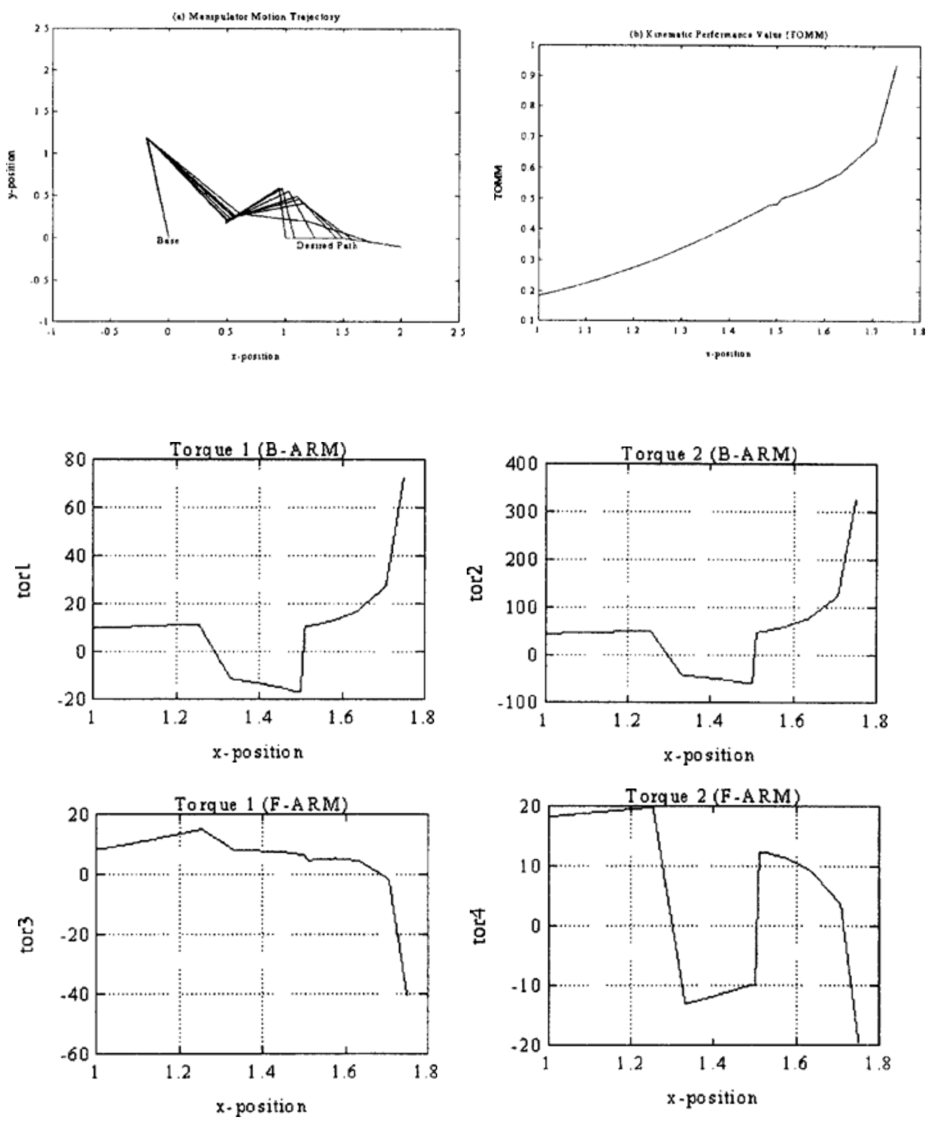


Fig. 4 Joint torques variation, $\gamma=0.1$,
torque limits : $-100N \cdot m \leq \tau_1,\ \tau_2 \leq 100N \cdot m,\ -30N \cdot m \leq \tau_3,\ \tau_4 \leq 30N \cdot m$

efficiency for generating Cartesian acceleration, and (b) the configuration of F-ARM is poor for resisting static forces normal to the surface.

The values of TOMM indicate this situation by a rapid increase as shown in Fig. 4(b).

4. This situation is resolved globally by the kinematic control of the elbow in the beginning of the second task segment, which guides the F-ARM to have a good configuration for the next task segment. This is demonstrated by Fig. 5. Note that two configurations are shown for the hand location (1.5, 0) : one configuration before elbow control, and the other after elbow control.

5. Using the F-ARM primarily allows the task to be executed with low power consumption and with high position control accuracy (Lee and Lee, 1990). Comparing Fig. 3 (using only the B-ARM) and Fig. 5 (using the F-ARM primarily with the elbow control in the beginning of the second task segment), it is clear that using the F-ARM for high-frequency motion is good for saving energy.

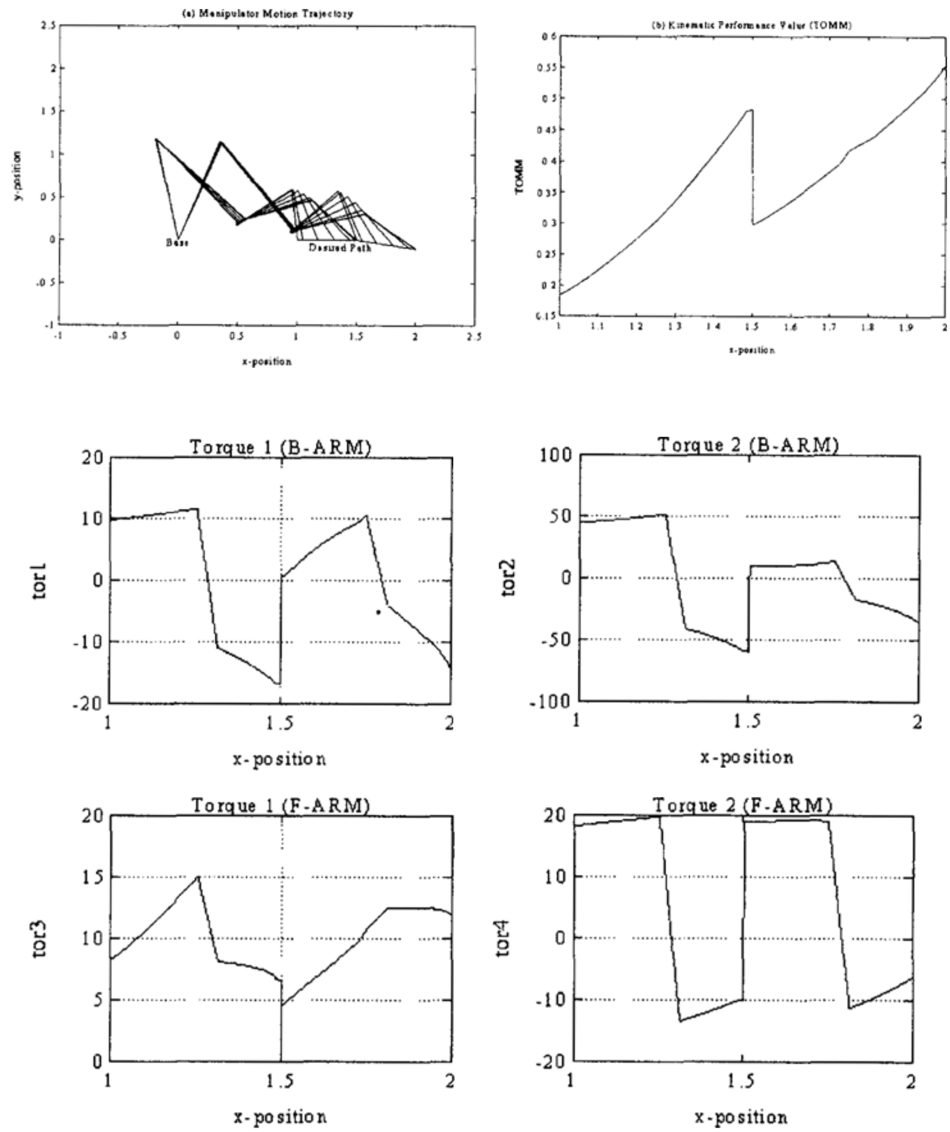


Fig. 5 Joint torques variation with elbow control at $x=1.5$, $\gamma=0.1$, torque limits : $-100N \cdot m \leq \tau_1,\ \tau_2 \leq 100N \cdot m,\ -30N \cdot m \leq \tau_3,\ \tau_4 \leq 30N \cdot m$

7. Conclusion

Dynamic characteristics of a decomposed-redundant manipulator have been utilized for the efficient task execution. That is, a task represented as a desired Cartesian acceleration is distributed to local arms forming a decomposed-redundant manipulator depending on their local dynamic characteristics. Through coordination between the local arms, the avoidance of joint torque limits and the reduction of power consumption are achieved.

It was also shown that assigning more of the task to the F-ARM is effective for both minimization of power consumption and accurate position control (Lee and Lee, 1990). However, the effects of heavy load to the F-ARM, which has relatively small workspace, may drive the F-ARM near to singular configurations where the required joint torques are very large. The dynamic characteristics (C. F. E and C. A. E) of the manipulator do not directly represent kinematic performance, for example, manipulability and static force resistivity. Therefore, kinematic control of the elbow can be adopted to adjust the configuration of the F-ARM to be versatile and suitable for a given task using TOMM as a performance index. It is suitable to control the elbow kinematically during the constant velocity phase or during the low acceleration phase, in which joint torque limits and power consumption are not dominant factors to be considered for efficient task execution. The elbow control can also be used to execute a subtask specified at the elbow, for example, obstacle avoidance.

In fact, this scheme can be used as a general tool to achieve cooperative control of macro/micro manipulator system (Sharon et al., 1988), vehicle/manipulator system (Yoerger and Slotine, 1987), or a reconfigurable manipulator systems (a serially linked manipulator system in which the F-ARM can be dynamically replaced for a specific task). The multiple task point control scheme is a promising method for optimal control of redundant robot arms and confers many advantages in real-world applications.

References

Book, W. J., 1985 "New Concept in Lightweight Arms," *The 2nd Int. Symp. of Robotics Research*, MIT press, pp. 403～405

Chang, P. H., 1986 "A Closed-Form Solution for the Control of Manipulators with Kinematic Redundancy," *Proc. IEEE Int. Conf. Robotics and Automation*, pp. 9～14

Horak, D.T., 1984 "A Simplified Modeling and Computational Scheme for Manipulator Dynamics," *Journal of Dynamic Systems, Measurement and Control*, Vol. 106, pp. 350～352

Hollerbach, J. and Suh, K. C., 1987 "Local Versus Global Torque Optimization of Redundant Manipulators," *Proc. Int. Conf. on Robotics and Automation*, Raleigh, North Carolina, pp. 619～624

Khatib, O., 1987 "Unified Approach for Motion and Force Control of Robot Manipulators : The Operational Space Formulation," *IEEE Jour. of Robotics and Automation*, Vol. RA-3, No. 1, pp. 43～53

Klema, V. C. and Laub, A. J., 1980 "The Singular Value Decomposition : Its Computation and Some Applications," *IEEE Trans. on Automatic Control*, vol. ac-25, No. 2, pp. 164～176

Lee, Sukhan and Lee, Jang M, 1988 "Task-Oriented Dual-Arm Manipulability and Its Application to Configuration Optimization," *Proc. 27th IEEE Conf. on Decision and Control*, Austin, TX

Lee, Sukhan and Lee, Jang M., 1990 "Multiple Task Point Control of a Redundant Manipulator," *Proc. IEEE Int. Conf. Robotics and Automation*, pp. 988～993

Lunde, E., Egeland, O. and Balchen, J. G., 1987 "Dynamic Control of Kinematically Redundant Robotic Manipulators," *Modeling, Identification and Control*, Vol. 8, No. 3, pp. 159～174

Li, Z. and Sastry, S. 1988 "Task Oriented Optimal Grasping by Multifingered Robot Hands," *Proc. IEEE Int. Conf. on Robotics and Automation*, Raleigh, NC, pp. 389～394

Liegeois, A., 1977 "Automatic Supervisory Control for the Configuration and Behavior of Multibody Mechanisms," *IEEE Trans. On Systems, Man, and Cybernetics*, Vol. SMC-7, No. 12, pp. 868～871

Maciejewski, A. A. and Klein, C. A., 1985 "Obstacle Avoidance for Kinematically Redundant Manipulator in Dynamically Varying Environments," *The Int. Jour. of Robotics Research*, Vol. 4, No. 3, pp. 109～117

Nakamura, Y. and Hanafusa, H., 1987 "Optimal Redundancy Control of Robot Manipulators," *The Int. Jour. of Robotics Research*, Vol. 6, No. 1

Sharon, A., Hogan, N. and Hardt, D. E., 1988 "High Bandwidth Force Regulation And Inertia Reduction Using a Macro/Micro Manipulator System," *Proc. IEEE Int. Conf. on Robotics and Automation*, pp. 126～132

Tzafestas, S. et al., 1988 "Robot Model Reference Adaptive Control Through Lower/Upper Part Dynamic Decoupling," *Journal of Intelli-*

gent and Robotic Systems, No. 1, pp. 163~184

Whitney, D. E., 1969 "Resolved Motion Rate Control of Manipulators and Human Prosthesis," *IEEE Trans. Man-Machine Syst.*, Vol. MMS-10, pp. 47~53

Yoshikawa, T. "Analysis and Control of Robot Manipulators with Redundancy," *The 1st Int. Symp. of Robotics Research*, MIT press, pp. 735~748

Yoerger, D. and Slotine, J., 1987 "Task Resolved Motion Control of Vehicle-Manipulator System," *Int. Jour. of Robotics Automation*, vol. 2, no. 3, pp. 144~150